# Analyzing Signals
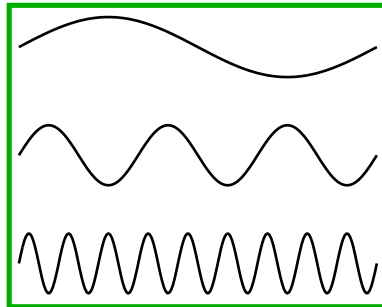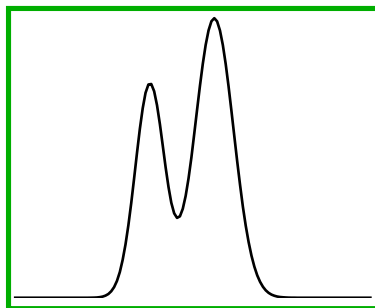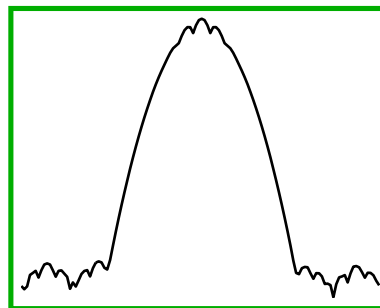
## Fourier transform
- frequency content
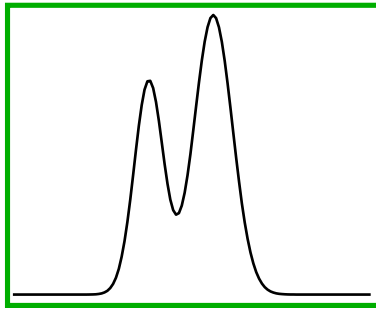- linear combination of $\sin(\omega t)$ and $\cos(\omega t)$

## Spectrum



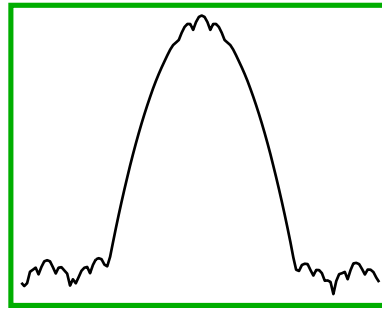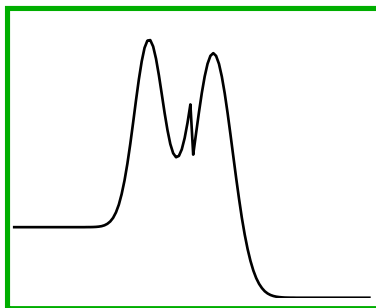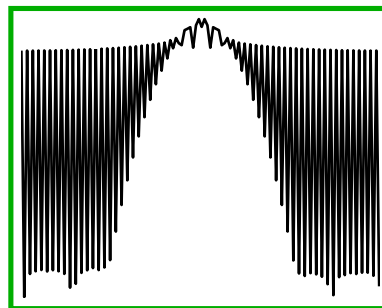spatial domain       frequency domain

# Spectrum

| spatial domain | frequency domain |
|:---:|:---:|

# Spectrum

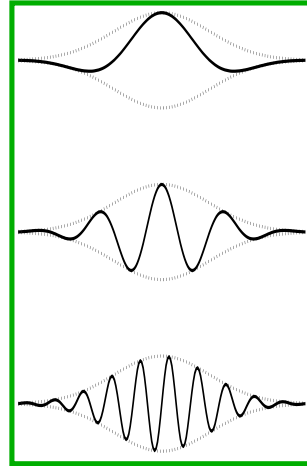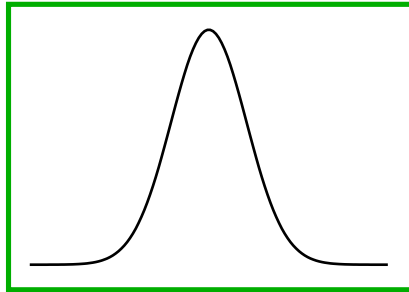| spatial domain | frequency domain |
|:---:|:---:|

# Localized Analysis

## Gabor (1940)

- time frequency analysis
- windowed Fourier transform

# Gabor Transform

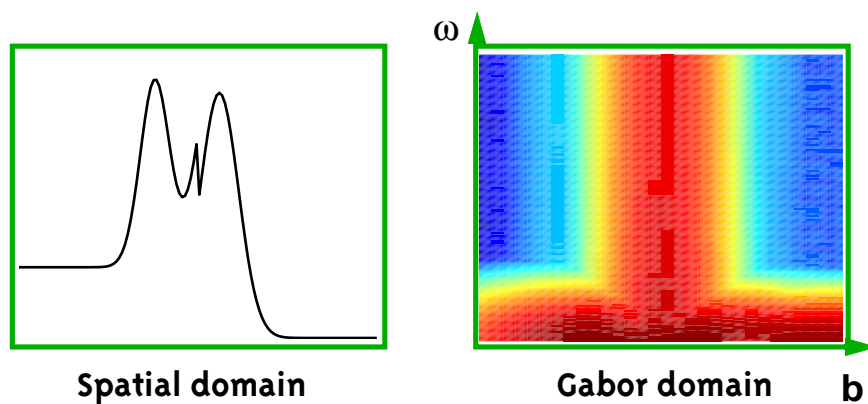## Find

- frequency $\omega$ in the vicinity of **b**

function to analyze

$$F(b,\omega) = \int f(x)g(x-b)\sin(\omega x)dx$$

window function at **b**

at frequency $\omega$

# Gabor Transform

$\omega$

**Spatial domain**

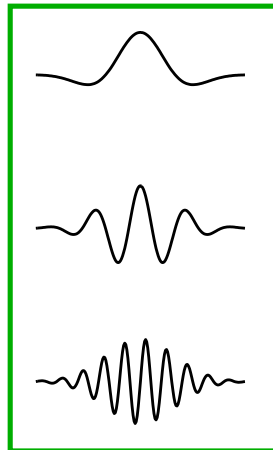**Gabor domain**   **b**

# Gabor Transform

## Problems

- discrete version very difficult to find
- no fast transform
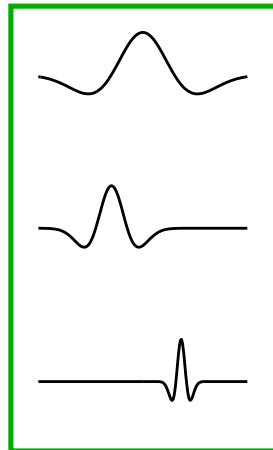- fixed window size!

## Solution

- large windows for low frequencies
- small windows for high frequencies

# Gabor Transform



Gabor bases          Wavelet bases
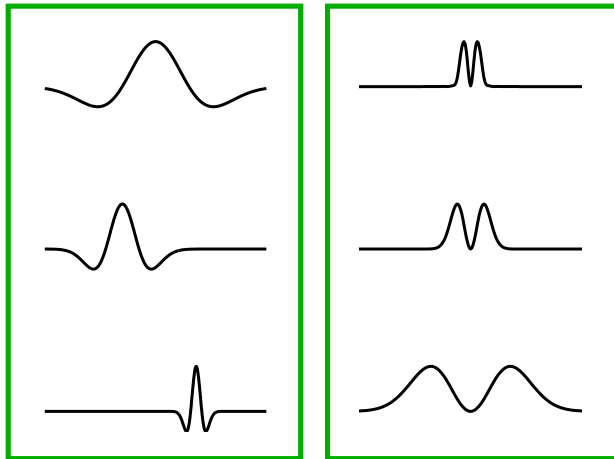
# Wavelets

## Translates and dilates of one function

$$\psi\left(\frac{x-b}{a}\right)$$

## Mother wavelet

- local in space
- local in frequency
    - smooth: no high frequencies
    - integral zero: no low frequencies

# Wavelets



Wavelet bases       Spectra

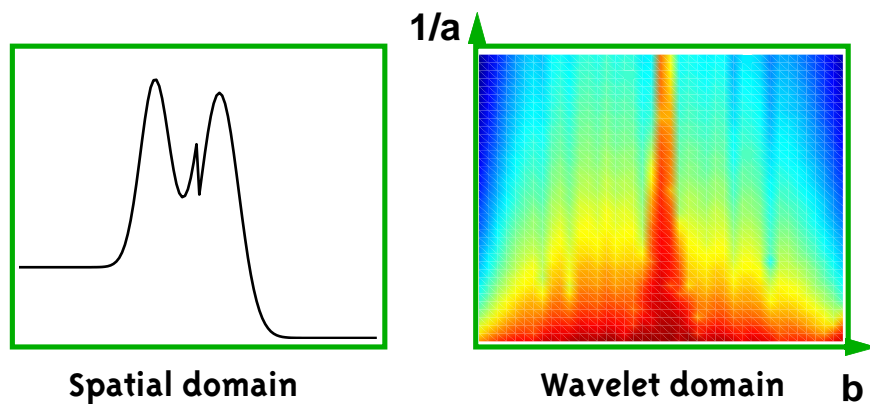# Wavelet Transform

## Find

- scale **a** at location **b**

function to analyze

$$F(a,b) = \int f(x)\psi\left(\frac{x-b}{a}\right)dx$$

Wavelet

# Wavelet Transform



**Spatial domain**

**Wavelet domain**

1/a

b

## Summary

### Fourier analysis
- global frequency properties

### Picking out local phenomena
- windowed Fourier transform: Gabor

### Wavelets
- window varies with frequency

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## Making it Practical

### A simple example
- Haar transform

### Building more powerful transforms
- Lifting scheme

### Generalizations
- making it work on general domains

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